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Dimensional Measurements

7.1 INTRODUCTION

Product development depends on exchanging information that at its root is quantitative and intended to represent product geometry. Earlier chapters are based on the assumption that the numbers needed to implement the design techniques were available and of sufficient quality to support the development process. To be useful, these numbers must be a valid representation of the physical features they are intended to portray. How the validity of this information might be assessed is one of the topics of this chapter.

Along with concerns relating to validity, the dimensional information must also be sufficiently accurate to support manufacture of interchangeable products. Without accuracy, the results will be suspect and may lead to undesirable or inappropriate product decisions. The development team must make measurements to obtain this dimensional knowledge. These measurements may support early stages of product development or may be useful for process development and control, final inspection, audit, or calibration purposes. For such applications to effectively contribute to the development process, the underlying theory of dimensional measurement (i.e., dimensional metrology) must be considered in implementing good measurement practice. Three major elements of good practice are covered in this chapter: general measurement theory, underlying statistical concepts, and measurement planning.

7.2 MEASUREMENT THEORY

The foundation of the measurement process is set on scientific principles, which if entirely understood and controlled would eliminate the need for statistical analysis. The process would yield a unique and valid number. However, the physical world is not quite this orderly; even the act of defining what measurement is to be taken is fraught with complexities not easily resolved. No single value emerges from the measurement process even when attempts are made to repeat the procedures under identical conditions.

7.2.1 The Measurement Model

To put the discussion on a logical footing, a model aids in defining terms and illustrating the concepts. One simple model of the measurement process is the following:

$$\text{Measurement} = \text{true value} + \text{error of measurement}$$

The true value is the theoretically correct or perfect value for the particular measurement being taken. The model implies that there is complete definition of the measurement and physically realizable procedures to obtain it. This is discussed further in Sec. 7.2.2. The true value is presumed to be deterministic in nature, having a unique value derived from its definition.

The error term can be expanded to include the following:

$$\text{Error of measurement} = \text{random error} + \text{systematic error}$$

The error terms are random variables based on probabilistic relationships governing the values they may assume. Consequently, the error cannot be characterized by a single, constant value; its nature must be captured by a statistical distribution. Furthermore, if the measurement results are to have any predictive value, the error terms must come from a process that is in a state of statistical control. A formal plan is needed that provides the necessary definitions of the measured quantity and corresponding methods and procedures to establish control. Without the measurement planning process to establish definitions and procedures, there can be no stable process and, as a consequence, no useful results.

With both deterministic and probabilistic elements providing the foundation for the model, the measured value must be described with a statistical distribution. The result of the measurement procedure is a range of values purporting to capture the true value, although some values are more likely to occur than others.

7.2.2 True Value

To clarify what is meant by “true value,” we can illustrate subtleties of the underlying idea by a simplified example. An engineering drawing ([Figure 7-1](#)) provides

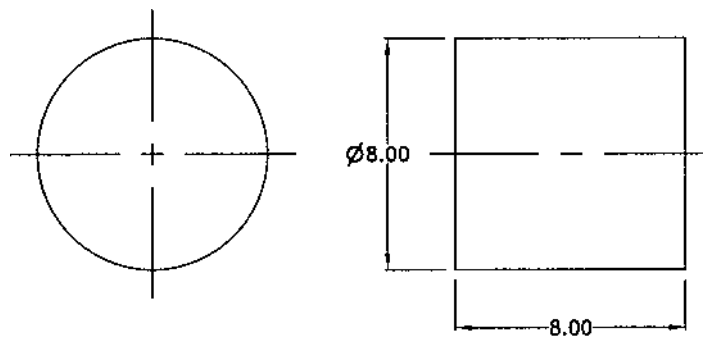


FIGURE 7-1 Engineering drawing of simple part.

a product definition for a cylindrical pin. The physical measurement is represented by the cross-section in Figure 7-2. This illustration is one possible cross-section of many that might result from the process used to gather the data set. The figure displays the measurement results by magnifying the radial deviations from the perfect form envisioned by the engineering drawing.

The balloons in Figure 7-2 identify possible methods of describing the “size” of the cylinder. The first balloon is attached to the profile, as seen by the inspection machine, and illustrates via magnification the form deviations present in a production part. The remaining balloons describe methods that might be used to attach values to the “size” of the cylinder, none of which guarantees achieving the same numerical value or functional outcome as other methods.

Balloon 1 represents the actual profile with radial magnification.

Balloon 2 shows a two-point measurement taken over apparent high points.

Balloon 3 shows a circle whose diameter is based on an arithmetic average.

Balloon 4 illustrates the minimum circumscribed circle for the contour.

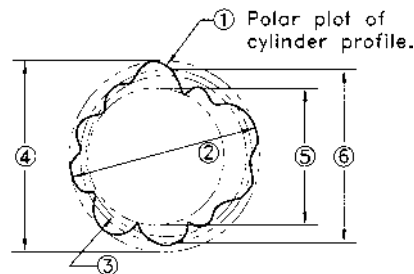


FIGURE 7-2 Various definitions of size.

Balloon 5 is the maximum inscribed circle.

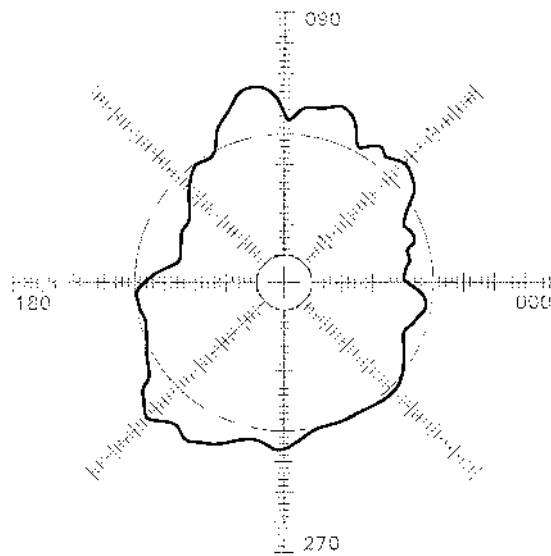
Balloon 6 is the fitted circle that might result from a mathematical algorithm such as a least-squares fitting.

From the figure it is apparent that no unique value may be attached to the “size” of this part. The mathematical complications of selecting a definition of size are further compounded by a variety of size definitions (e.g., actual size, actual local size, actual mating size) contained in the Y14.5M and Y14.5.1M standards.

The designer had a functional specification in mind when the drawing in [Figure 7-1](#) was created. This functional specification may be better served by one of the “size” definitions here than by others. Hence, the specification must include a verification procedure to provide complete functional definition. The procedure makes explicit the choice of a size definition appropriate to the functional requirements. As an example, if a particularly loose fit is the criterion the designer used, then the two-point measurement over the high points—a local size—may be an acceptable definition. If a more precise fit is the intended design goal, the minimum circumscribed cylinder—the actual mating size—could fill the requirement, remembering that a three-dimensional fit will need a circumscribed cylinder rather than a circle for verification. Either of these might accomplish the implied functional outcome in a specific situation. However, the indiscriminate choice—or no choice—of measurement method could result in an unintended functional outcome, higher costs incurred in verifying compliance with the product definition, and ultimately unacceptable costs associated with functional failure of the product. The development team will have to decide which definition is operable within the design and economic context of the project.

An example can be used to illustrate the problem. [Figure 7-3](#) is a polar chart of an out-of-roundness measurement for a feature that provides a high-pressure seal. The original engineering drawing addressed the issue of form control only tangentially through an explicit size tolerance ($\pm .0005$ in.) on the feature. There was no specific geometric tolerance to control roundness. Measurement planning followed the implicit instructions in the engineering specification and did not check form characteristics. The selected measurement procedure resulted in too large an uncertainty to expose the form characteristics of the manufacturing process. The unintended consequence was failure of the assembly to produce the necessary seal because the design did not establish the functional definition of the geometry. Without complete functional specification, the planning process did not include verification procedures capable of recognizing an impending functional failure (i.e., out-of-roundness).

In theoretical terms, this example illustrates the “definition problem.” As can be seen, even a rather simple geometric shape such as the cylindrical pin can provide challenges to the designer of the product or measurement system. If these challenges were categorized, the two major ones would be the errors attrib-



ROUNDNESS .00144 IN

FIGURE 7-3 Polar chart of out-of-roundness measurement.

utable to the theoretical description of the measurement (i.e., the definition problem) while the remaining errors are placed in the category of variation in the actual measurements.

In reality, there can never be a complete definition of the measured quantity. Too much information is required to provide complete specification of the measurement. However, like many of the processes previously discussed in the book, the definition of the measurement (i.e., the measurand) can be refined to a point where it incurs an acceptable level of cost based on design requirements and economics of the situation.

The definition of dimensional measurement to be used in this book is expansive. It includes not only the obvious ideal geometry the designer chose but also the method of measurement and the executable procedures needed to acquire and analyze the data set. Thus, the definition of the measurement is fluid, taking on different forms depending on its position in and contributions to the overall product development process. In a specific case, the planning process might lead to using a micrometer to gather two-point measurements for process control, digital sampling using a coordinate measuring machine for functional inspection, or any of a number of other techniques capable of evaluating size and other geometric characteristics. Which is appropriate is a question that must be answered by the intended use of the measurement.

7.2.3 Error and Uncertainty

In the context of physical experimentation, error means “the inevitable uncertainty that attends all measurements” (Taylor, 1982). In the measurement model, error is made up of fluctuations in measurements that may take the form of either random effects or systematic errors.

The random effects may be demonstrated by again referring to the example in [Figure 7-2](#). Each two-point measurement taken at the same cross-section of the pin will yield a different value. Furthermore, each cross-section on the pin will also give differing measurements as the micrometer is repositioned around the perimeter of the part. The random effects can be quantified by using statistical methods to describe the dispersion of values that attends the measurement of the pin. This dispersion of values provides an estimate of the error and is what is meant by “uncertainty.” The statistical property used to describe the dispersion (or variation) of the measured quantity is the standard deviation. This is the quantity ultimately used to characterize the uncertainty of a measurement, although not quite in its initial form.

The systematic component of the error (e.g., bias in measuring systems) is the “error that will make our results different from the ‘true’ values with reproducible discrepancies” (Bevington and Robinson, 1992). Statistical data analysis cannot identify this type of error. A statistical analysis that yields the mean and the standard deviation would be silent on the existence of systematic error. Using other measuring methods that by agreement are free from bias and comparing the resulting values with the values of interest might deal with this quandary. Another less effective technique would be to use other measuring devices and see if the values compare. The latter method may support the contention that the first device yielded unbiased results but does not prove it.

7.2.4 Precision and Accuracy

Two additional terms, precision and accuracy, are bound together much as error and uncertainty are. The term “precision” may also have a connotation similar to that of uncertainty.

Precision is the variation present in a set of measurements; it excludes the question of whether they are reasonable representations of the “true value.” As mentioned, the usual choice of a statistic to describe this variation is the standard deviation. Consequently, when the standard deviation is quoted, the precision of the measurement process is being described. Continuing along this logical path, when the precision of a measurement is cited, one is also describing the uncertainty of the measurement. Actual statements of uncertainty are more involved, using a coverage factor to expand the standard deviations into a form acceptable under standardized techniques for uncertainty expressions.

The accuracy of a measurement is the closeness of its value to the true

value. The model in this book implies that the true value is known, which is not possible. Much about the physical world is not yet understood, making it impossible to completely prescribe a measurement. Finding the “true value” of a measurement could rapidly evolve into a scientific exercise of suspect economics. In some instances there are conventional values that are accepted as the true value. For most manufacturing situations, a particular level of refinement (i.e., an acceptable level of uncertainty) must be explicitly chosen to implement a measurement procedure. The measurement planning process, including definition of the measurement, takes place within the development process, not in isolation. Choices are made that encompass physical principles and economics, the latter requiring detailed measurement planning.

7.2.5 Precision, Bias, Accuracy—An Illustration

Precision, bias, and accuracy are frequently used to characterize data but are applied in less than precise ways. The preceding sections have given more concise definitions of these terms, but an illustration should serve to clear up any lingering doubts as to their meanings. The classical illustration is that of target shooting with a variety of patterns serving to illustrate the meaning of each term and show the connections that must be made when choosing appropriate terminology.

The example here (Figure 7-4) uses a positional tolerance zone as the target and a single hole as the measured feature. Repeated measurements are made on this hole. Each set of measurements is made in a different setup, resulting in a specific pattern representing that set of observations. Each of these figures is assumed to have a different set of factors that affect uncertainty and provide the influences that result in the observed patterns.

The first measurement process [Figure 7-4(a)] shows a pattern with high dispersion that misses the specified tolerance zone. This pattern could result from loosely defined procedures implemented over a fairly long time span. The part is also set up on the wrong datum, introducing a bias represented by the pattern’s being centered away from the target. The wide dispersion may be the result of swings in environmental factors (e.g., time-dependent temperature changes) that are not controlled sufficiently well. In this book’s terminology, this pattern illustrates measurements that have high bias and low precision.

The second figure [Figure 7-4(b)] might show the same measurement process with the part correctly located on specified datum features. With no additional improvements in control of the environmental factors, the dispersion is still similar to the first measurement process, but the bias has been eliminated. This pattern shows an example of low bias and low precision.

Next, the part is measured in a well-planned process [Figure 7-4(c)]. But the inspector has located the part from the wrong datum surface as in the first example. This mistake in location has introduced bias. However, the measure-

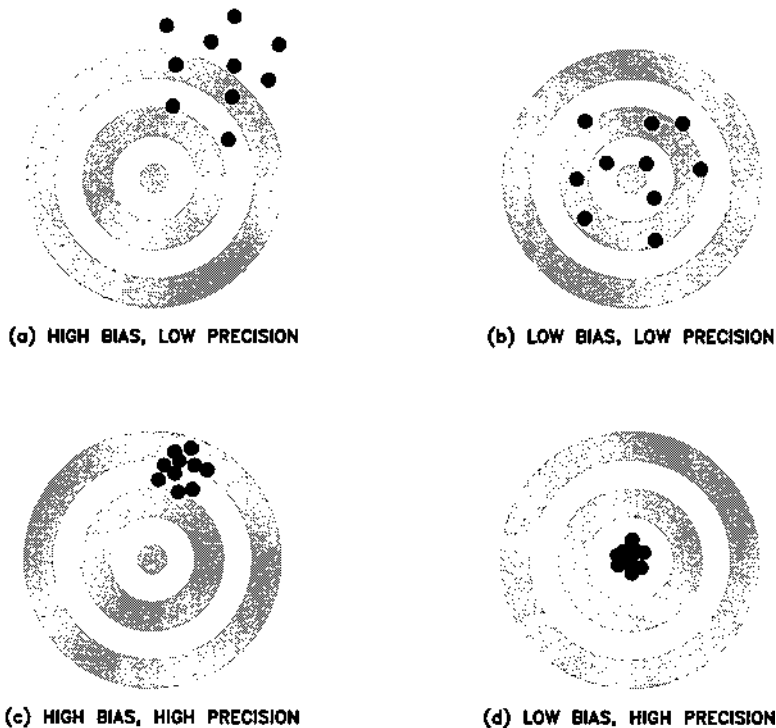


FIGURE 7-4 Bias and precision in measurement results.

ment planning process has reduced the dispersion by effectively reducing error contributors such as temperature effects. Hence, the result is a process that is highly biased but has high precision.

The last example [Figure 7-4(d)] demonstrates what happens to the measurements when all elements of the process are prescribed. The inspector now follows the measurement plan and achieves outstanding results. The part is correctly located in the specified datum reference frame, and all identified error contributors are controlled from measurement to measurement. The pattern now exhibits both low bias and high precision.

All these examples have used the words “bias” and “precision.” In the world of interchangeable manufacturing, one is also concerned with being on the target. This involves accuracy. Looking back at the four patterns used to illustrate bias and precision, the last example [Figure 7-4(d)] demonstrates what might be called an accurate measurement process. It is a process in a state of statistical control that exhibits low bias and high precision. Any other combination of bias

and precision does not guarantee accuracy. A process with high variability [i.e., the low precision of [Figure 7-4\(b\)](#)] makes it difficult to say that one is on target. A process exhibiting high bias is obviously not on target. Even if the latter process had high precision, it still does not provide accurate results. Only the combination of high precision and low bias allows a process to be described as accurate.

7.3 STATISTICAL TECHNIQUES

In the past, the gagemaker's art was sufficiently advanced relative to the manufacturing process to produce gages that utilized a small portion of the product's working tolerance. Gages could be obtained with sufficiently small errors, allowing economical control of the product. In recent years, manufacturing techniques have been developed that have process capabilities exceeding the companion measurement processes. The consequent disparity between what can be made and the ability to measure it complicates the situation. This disparity in abilities has not always been the case, nor is it the major consideration in all instances of current manufacturing practice. However, it is no longer rare to see the limitations of measurement capability having an impact in applications of high manufacturing accuracy. In such situations, it is quite conceivable that the measurement process will only recognize extreme (catastrophic) failures and only then take corrective action.

Another area where the relative capabilities of the manufacturing process vis-à-vis the measurement process causes concern is in a measurement chain where traceability is required. If traceability is based on a traditional 10% rule, each of the elements in the traceability chain has precision requirements that may rapidly exceed the capability of commercially available techniques and equipment. Consequently, it would be beneficial if methods could be found to increase the effectiveness of the measurement process without incurring tremendous cost for more accurate equipment and environments.

As it happens, there are both statistical and planning techniques that can fill the void if understood well and used appropriately. We can improve the effectiveness of the measurement process by taking the average of multiple measurements (i.e., the arithmetic mean) and using this as the estimate of the measured quantity. Such a statistical practice can increase the precision of the measurement beyond levels found in procedures dependent on a single measurement. This is accomplished by taking advantage of underlying statistical theory.

7.3.1 Statistical Concepts

The statistical techniques of use in measurement applications rest on three general ideas. The first is that the random sampling process generates values of a measurement that are repeated from sample to sample, creating a pattern that is relatively

constant. One such pattern is the bell-shaped, normal distribution. Second, the sampling process provides a useful estimate—an unbiased estimator in statistical terms—of the quantity being measured. Third, the precision of the estimate is governed by the size of the sample, which is controlled by the measurement planner.

Measurement techniques rely on two different types of statistical distributions: the distribution of the individuals and the distribution of the averages. The former is the distribution of individual measurements, in theory the population of all possible measurements of the specific measured quantity that may result from the measurement procedures. The distribution of averages is the distribution of the arithmetic means that may be calculated for all possible samples of a specific size drawn from the individuals' population. The statistical name for this last description is the "sampling distribution."

The theory that underlies the sampling process has a name, the *central limit theorem*. While the intent of the book is not to teach statistics, the consequences of the central limit theorem can be illustrated graphically using [Figure 7-5](#). In this figure, the top row of graphs shows three distinctly different populations (i.e., the individuals). Below each of the population distributions are three examples of a sampling distribution drawn from the respective population. Each sampling distribution is created by repeatedly drawing a sample of a set size from the population and calculating the sample's average. These averages are graphed and used to represent the sampling distributions.

The initial interest is focused on the location of the means of the population and its companion sampling distributions. In addition, the shape of the sampling distribution demonstrates the critical effect that the sampling process can have on estimation of the measured quantity.

Looking at the graphs, one can see that the three sampling distribution means related to a particular population example are located in the same position along the horizontal axes as the mean of the parent population. This graphical result demonstrates what the central limit theorem predicts: The mean of a sample is a good estimate of the unknown population mean. In fact, the theorem indicates that in the extreme (i.e., very large or infinite sample sizes) the mean of the population and the mean of the sampling distribution should be the same. Hence, the mean of the sample measurements can provide a good approximation (i.e., a good estimate) of one's desired measurement even without resorting to large sample sizes. It does not provide the true value, but it is reasonable to expect the calculated sample mean to lie in the neighborhood of the true value if measurement planning was done correctly. One may choose the size of the neighborhood.

Comparing the relative shapes of the sampling distributions also yields a useful fact. As seen in the figure, as the sample size is increased, the variation about the mean of the sampling distribution is reduced. From a statistical stand-

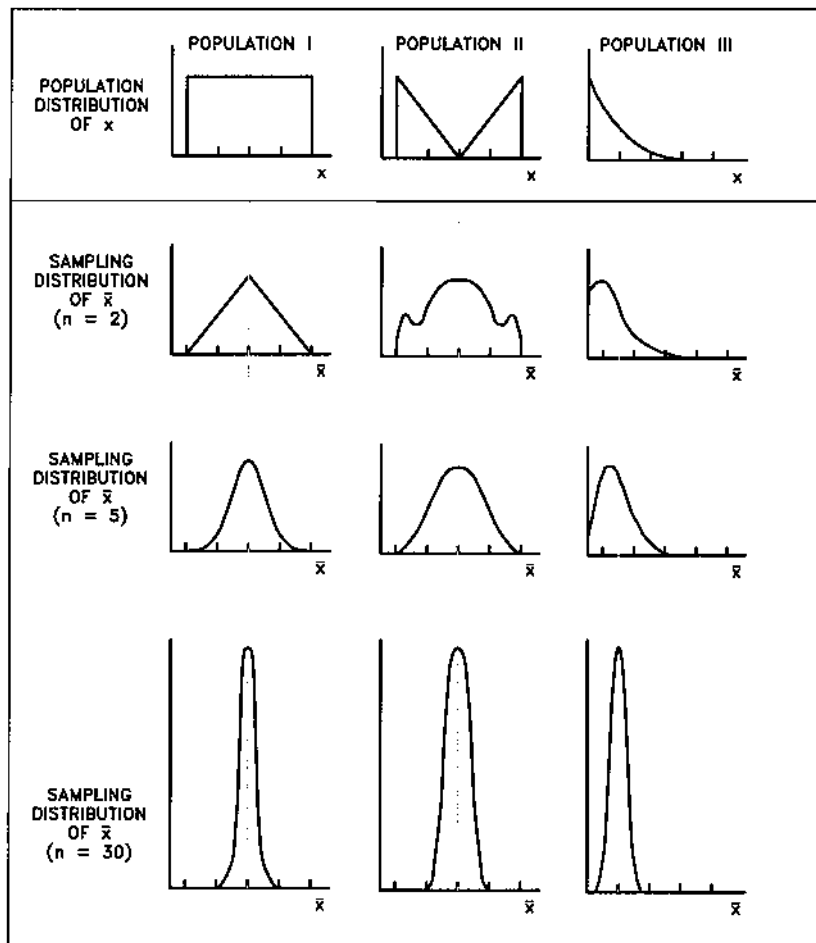


FIGURE 7-5 Consequences of the central limit theorem.

point, one gets better estimates of the mean (less variation) as the sample size increases. That this is reasonable can be demonstrated by carrying the sampling to a logical extreme. If the entire population is sampled (called a *census*), then there can be no variation in the estimate of the mean of the population because the true value has been obtained. While no census can be taken of the theoretical population of our measurement, the desired effect can be achieved by taking multiple measurements of the quantity of interest and using the average of this sample as the estimate of the true measurement. The variation about the estimate

of the mean can be reduced (i.e., one can get a better estimate), but it can never be eliminated since there is no finite population of measurements from which the census may be taken. Furthermore, the cost of extremely large samples taken in an attempt to drive the variation to zero would be prohibitive.

The connection between the shape of the population and the shape of the sampling distribution should be made explicit. The central limit theorem predicts the shape of the sampling distribution to be a normal (Gaussian) distribution as the sample size becomes larger. This is also illustrated in the figure (Figure 7-5). As the sample size becomes large (the lower set of diagrams), the apparent shape of the distribution is that of a normal distribution. With the highly skewed Population III, this effect takes a little longer (i.e., larger sample size) to be achieved, but once one gets near a sample of size 30, even a sampling from the skewed population takes on the shape of the bell-shaped curve. Knowledge that the shape of the sampling distribution is normal allows one to make reasonable estimates of the confidence that one can have in the results of the measurement process.

There are limitations to the application of the central limit theorem. Knowing the shape of the sampling distribution tells nothing about the shape of the parent population from which the sample was taken. If the distribution of the individuals must be used for some design purpose such as assembly-level tolerancing, taking a sample and calculating its mean and standard deviation does not provide information about the population shape (i.e., the distribution). If the population shape is to be estimated with sufficient confidence to predict probabilities associated with assemblies composed of the individual features that the measurements represent, additional statistical investigation is needed.

7.3.2 Random Uncertainties

Returning to the measurement model here, random fluctuations are included in each of the measurements taken. The accompanying graph [Figure 7-6(a)] illustrates one source of these fluctuations by showing a statistical distribution (represented as a histogram) associated with the raw measurements. These are the uncorrected (i.e., raw) measurements taken during the actual inspection process and representing the “individuals” discussed in Sec. 7.3.1. Such fluctuations are the consequence of a host of possible uncertainty contributors discussed in the next chapter.

At this level, the information from the measurement process is a range of values that result from a sample of some set size—represented by the letter n —deemed appropriate by the inspector or the measurement planner. In theory, an infinite number of samples of this size may be drawn from the population of measurements that could be taken on this specific workpiece feature. As a consequence, the distribution shown in the figure is only an estimate of the actual population of the measurements. With care, it is a good estimator, but it is still an estimate and not the actual population distribution. So one has a little—or a

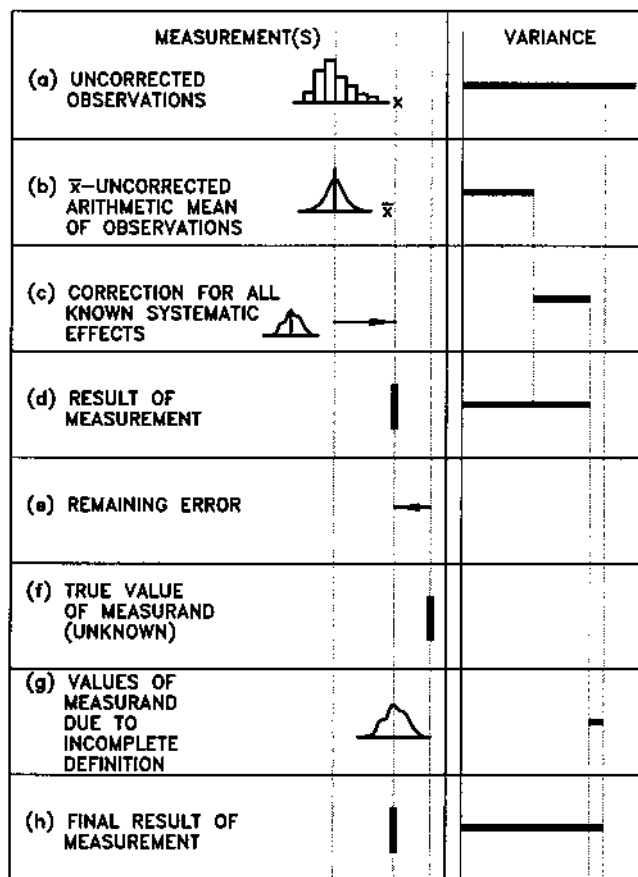


FIGURE 7-6 Illustration of measurement values, error, and uncertainty. (Follows ISO guide to the estimation of uncertainty in measurement (GUM).)

lot—of uncertainty introduced at this first stage of the process. An acceptable level of uncertainty is present if care is taken in defining the measurement; an unacceptable level of uncertainty is present if no planning is done and the measurement process is entirely uncontrolled.

Following convention, descriptive statistics are calculated for the data taken and a graphical display generated. These statistics typically include the arithmetic mean and the standard deviation. For the raw data, the uncorrected arithmetic mean [Figure 7-6(b)] is still shown in the figure with a distribution rather than a unique value, indicating the existence of uncertainty even in this value. The uncertainty arises since another sample of the measurements of the same size as the first sample will give a different value for the mean. If done correctly, the

second mean will be close to the value from the first sample but will not match exactly.

Additional graphical information is added to [Figure 7-6](#) as the right-hand graphs. They illustrate the variation in the measured values and derived quantities shown in the left-hand graphs. Such variation is represented mathematically by the standard deviation (symbolically shown as σ) or its equivalent, the variance. For the raw measurement values, the variation is the standard deviation of the sample measurements (the individuals). Once the uncorrected arithmetic mean is calculated, the statistics represent values associated with the sampling distribution and the central limit theorem. Thus, the variation for the sampling distribution becomes the population standard deviation divided by the square root of the sample size, statistically referred to as the *standard error* of the mean. The important thing to note is that the variation of the sampling distribution is less—the central limit theorem at work—than the variation in the population. This will give a tighter range of values in which the true value is likely to reside.

At this juncture, it should be apparent that the fluctuations the statistical distributions in the figure show are mathematically portrayed by the standard deviation. These values represent uncertainty in measurement. Other sources of uncertainty enter into the measurement process, but at this stage of the discussion it is important to reinforce what the term “uncertainty” represents.

7.3.3 Systematic Uncertainty

Continuing with the graphs in [Figure 7-6](#), the mean of the raw data is adjusted [[Figure 7-6\(c\)](#)] for any systematic effects (i.e., bias in the measuring system). One such systematic effect could be a known temperature differential between the temperature of the workpiece and the reference temperature. While this correction might be treated as deterministic, it is likely there is uncertainty in the value of the coefficient of thermal expansion that makes a unique value unavailable. Uncertainty is just as likely in other corrections. In the figure, this underlying uncertainty associated with correction values is modeled by another, albeit smaller, statistical distribution.

It can be logically argued that the corrections can be treated as unique values (i.e., deterministic) rather than following statistical distributions. Such a decision can be based on the systematic component’s being small by comparison to total process uncertainty. This is another manifestation of the refinement concept introduced early in the text. The degree of refinement necessary to achieve the project’s uncertainty budget is one of the many decisions that must be made in bringing about economical production. The expense of determining the distribution underlying a systematic uncertainty contributor may be unwarranted if the correction makes a very small contribution to total uncertainty. Using a deterministic correction, the results are displayed in [Figure 7-6\(d\)](#).

7.3.4 Uncertainty in Definition

Preceding sections allude to the importance of the measurement definition. As pointed out, the definition given here encompasses more than normally considered when defining a measurement. It includes not only drawing representations of the ideal geometry associated with the measured feature but also methods and procedures to be followed in measuring the actual geometry and attaching a value to the measurement.

Reducing definitional uncertainty requires as complete a specification of the measurement, method of measurement, and procedures as is possible within the context of scientific knowledge and available economic resources. For example, one may choose a definition that does not incorporate all of one's knowledge, deferring to economic considerations to obtain a measurement that is "good enough." Under any circumstances, this argues for a formal measurement plan rather than defaulting to the individual inspector's judgment.

The definitional uncertainty is shown in the accompanying graph [Figure 7-6(g)] by the true value and a spread of values around it. If one cannot completely define what measurement is to be taken, then one cannot specify what the true value of this measurement should be. Uncertainty occurs because one cannot specify all the necessary details to achieve the true value. This may appear to be a purely academic point, but one very important conclusion should be drawn from this. There must be a formal measurement planning stage in every product development project to avoid introducing unexpected uncertainty into metrics used to make engineering and management decisions. If one does not know what is actually being measured and how the values are acquired, then the resulting information is useless. It is of no economic value.

7.3.5 Averages and Individuals—An Example

To show how the sampling process works to one's advantage, an example has been constructed using statistics taken from a real data set. In this case, a series of measurements was made on a composite sheet that had a thickness specification of 0.0430 to 0.0470 in. The sample measurements gave a mean thickness of 0.0449 in. and a standard deviation of 0.0014 in. These numbers are referred to as "sample statistics" to set them apart from the "population parameters" describing the theoretical population.

For the simulated example, a theoretical population that has the same mean and standard deviation found by the actual sampling process has been constructed. This theoretical population is modeled as a normal distribution, which may not be the actual case but serves to illustrate the general idea. This theoretical population will now be used to compare what happens when an individual measurement is used to represent a "true value" and, alternatively, what results if a sample of some size other than 1 is used to the same end.

The simulation uses a commercial statistics package to randomly generate 100 sample points. These are drawn from a theoretical population with the mean and standard deviation mentioned. The resulting sample of 100 points is shown in Table 7-1. Two graphical representations of this data set are shown in [Figure 7-7](#). The right diagram is a box-and-whisker plot and the left diagram is a histogram. The general shape of the histogram should reflect the normal distribution expected from the sampling process, and the sampling distribution's bell shape should be further enhanced since the assumed parent population is also normal.

The figure shows added information in the form of descriptive statistics calculated for the sample. The sample of size $n = 100$ gives a sample mean of 0.0450 in. and the standard error of the mean (the sample standard deviation divided by the square root of the sample size) of 0.0001 in., a very good estimate of the assumed population values.

If one uses the sample points in the table and looks at these values as

TABLE 7-1 Sample Data Set for $n = 100$

Sample	Thickness	Sample	Thickness	Sample	Thickness	Sample	Thickness
1	0.046292	26	0.046455	51	0.044547	76	0.045082
2	0.046062	27	0.04652	52	0.044762	77	0.046657
3	0.046067	28	0.043585	53	0.045516	78	0.044588
4	0.044504	29	0.043611	54	0.045105	79	0.045054
5	0.046633	30	0.046825	55	0.045577	80	0.043194
6	0.046460	31	0.043823	56	0.044179	81	0.046135
7	0.043619	32	0.045327	57	0.045472	82	0.045219
8	0.044431	33	0.045252	58	0.042553	83	0.044584
9	0.042625	34	0.044787	59	0.044596	84	0.047843
10	0.043773	35	0.044548	60	0.045728	85	0.045038
11	0.045980	36	0.041770	61	0.045949	86	0.044053
12	0.045277	37	0.047336	62	0.044467	87	0.046354
13	0.045569	38	0.045978	63	0.045089	88	0.046584
14	0.043898	39	0.045020	64	0.046888	89	0.047357
15	0.043275	40	0.047223	65	0.046738	90	0.047250
16	0.044570	41	0.044123	66	0.042545	91	0.042226
17	0.044681	42	0.045378	67	0.044482	92	0.041004
18	0.044939	43	0.047079	68	0.045068	93	0.045434
19	0.043421	44	0.044608	69	0.044453	94	0.041805
20	0.045920	45	0.044913	70	0.043977	95	0.046173
21	0.045358	46	0.047167	71	0.045192	96	0.047763
22	0.045394	47	0.043874	72	0.043530	97	0.045182
23	0.045073	48	0.044463	73	0.044715	98	0.045457
24	0.044076	49	0.045220	74	0.049442	99	0.043889
25	0.045394	50	0.044322	75	0.043238	100	0.044502

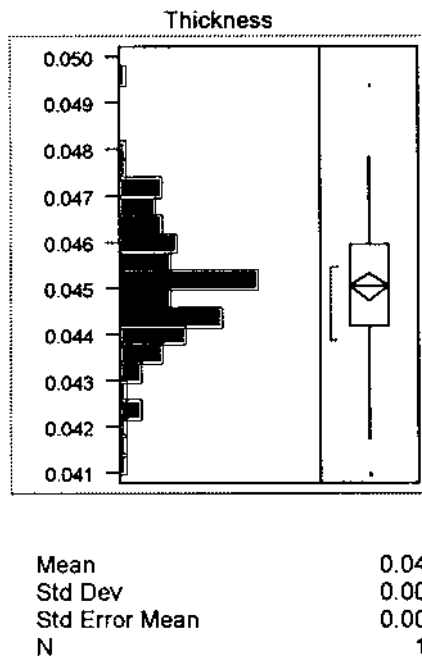


FIGURE 7-7 Graphical display of sample data set for $n = 100$.

individuals, a different story emerges. While the histogram shows that it is much more likely to take an individual measurement and find this value near the population mean of 0.0449 in., other possibilities may occur.

The two points on the box-and-whisker plot that stand out from the rest of the diagram are called *outliers*. These points are located at measurements of 0.0410 in. and 0.0494 in., well outside the specification limits. Now consider the case where a single measurement (an individual) is used to assess whether the process is operating at its target value. Further assume that the target is the mean of 0.0449 in. A sample point at either one of the extreme values would lead one to believe that the process is not centered properly. However, this little experiment has set the process on its target. So the process is centered where it should be although process variation is greater than desired. If centering adjustments are made based on one of the extreme values, the process mean, which is at target, will be moved. This will throw more production outside the specification limits than is already the case.

It is not necessary to take large samples to achieve the desired sampling effects. Again using the data points in Table 7-1, a random sample of size $n = 10$ is shown in Table 7-2. These 10 samples were randomly selected from the

TABLE 7-2 Sample Data
Set for $n = 10$

Sample	Thickness
1	0.046738
2	0.045082
3	0.044913
4	0.045073
5	0.044053
6	0.044939
7	0.045577
8	0.045277
9	0.045358
10	0.047763

Data taken from [Table 7-1](#).

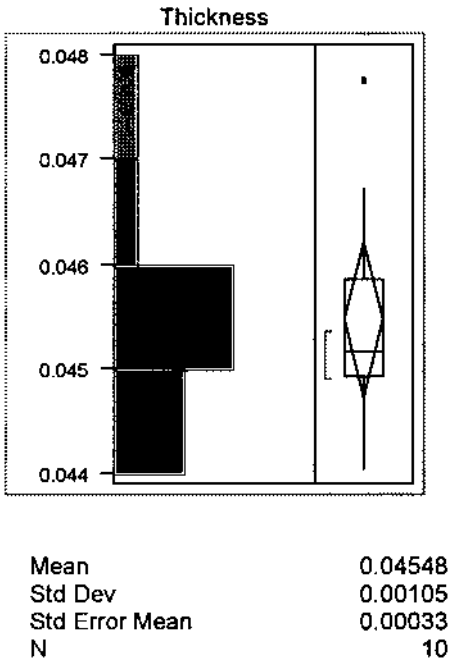


FIGURE 7-8 Graphical display of sample data set for $n = 10$.

original list of 100 measurements. The sample points are graphically displayed in [Figure 7-8](#) along with their sample statistics. As the central limit theorem predicts, the sample mean is still a good estimator of the population mean (0.0455 versus 0.0449). The standard error of the mean—the sampling distribution’s variation—is shown to be higher than the equivalent statistic from the larger sample (0.0003 versus 0.0001). However, remember that taking a larger sample can control this.

The key point is that the sampling process—with $n > 1$ —gives a better estimate of the population mean than the individuals and allows one to describe confidence in the estimate. The latter concern cannot be addressed if a single value is used as the estimator. Without a range of values to measure the variation, one cannot attach any confidence level to an estimate. A sample size of 1 is not of much value.

7.4 MEASUREMENT PLANNING

In product development the sequence of events starts by taking customer requirements and reducing them to product architecture. This stage of the process is examined earlier in the book. Product characteristics derived from the architecture—the interest here is primarily the geometry needed to implement customer requirements—must be translated into specifications. These specifications are then used for both product and process development. Measurements may be used for a wide variety of purposes in the development process: to determine specifications, monitor manufacturing process, and to verify whether design intent has been faithfully executed.

With engineering specifications in hand, a series of standards can be used to illustrate the logic that underlies the measurement planning process. In discussing this, the broad view common throughout the book is again taken. Measurement planning begins with specification of design intent on the engineering drawing and continues through the more obvious steps associated with selection of measurement methods and procedures.

The following material presents a formal measurement planning process. The need for planning can be highlighted by examples showing the relative effects of different elements of the measurement process. Based on the authors’ experience, only a small portion of the uncertainty of the measurement process is contributed by the measuring instruments. Other sources estimate that 10% to 20% of the uncertainty on the shop floor is attributed to the instrument calibration (Bennich, 1997). Larger effects are introduced by temperature (e.g., possibly three to four times larger) than by the calibration of the instruments. All the critical process elements (e.g., environment, setup, definition of the characteristic, etc.) must be managed to achieve a stable and controlled process. This state of

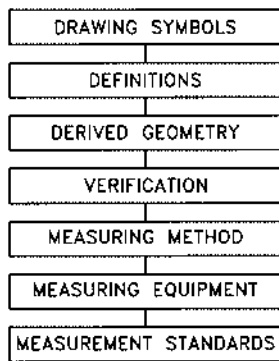


FIGURE 7-9 Chain of standards for characterizing product. (From ISO/TR 14638.)

statistical control, necessary to produce valid results, is obtainable only through a formal planning process.

The discussion can be structured using a chain of standards. This is the generic title applied to the group of standards that underlie the measurement process. The chain links are codified in an ISO technical report, ISO/TR 14638, and illustrated in Figure 7-9. The report deals with a plan that describes a geometrical product specification (GPS), allowing one to geometrically characterize the product. In the following discussion, the various chain links are referred to in more general fashion as steps. The intent is to convey more of the substance of the measurement planning process and less of the details of the GPS masterplan and underlying standards.

Reaching back to earlier ideas, the GPS standards support a design methodology and provide a communication aid for the dimensional planner to use. Without the rigor imposed by a formal process that uses the standards chain, one has an uncontrolled measurement process. This produces unusable values and moves design decisions downstream where design oversight is lacking.

A rather dramatic reason to encourage measurement planning is also cited by Bennich. In an examination of a particular country's firms, it was discovered that the typical uncertainty of measurement was greater than the drawing tolerance. This is essentially the same experience the authors have had with domestic firms and adds to the importance of the following discussion.

7.4.1 Functional Representation/Design Intent

Using the chain-of-standards analogy, the initial links are charged with representing the designer's functional intent. This is accomplished by standardized methods that indicate functional requirements on the engineering drawing. Selecting the Y14.5 standard as an example, one section in the standard deals with

symbology and syntax (step 1) used to specify geometric characteristics and dimensional requirements. Other sections of the same standard—and ASME Y14.5.1—then define the theoretical intent (step 2) of these drawing indications, providing definitions for specific types of part feature geometry and associated tolerance zones.

The chain-of-standards concept is a formal recognition of the definition problem highlighted earlier. For a measurement to be used in prediction and decision making, an adequate definition must be assembled that prescribes all necessary elements needed to achieve the desired level of accuracy. Each of these chain links contributes to building a definition supported by appropriate methods and procedures that underlie good measurement practice.

7.4.2 Derived Geometry

The next steps in the chain describe specific methods and procedures used to take the drawing's representation of design intent and apply it to the physical components. Step 3 recognizes that measurements may be taken in such a form that additional processing of the raw measurements is needed. This added processing derives the quantity that will be compared to the drawing specification. An obvious example in this area would be using the CMM to determine size and a host of other geometric characteristics. The CMM addresses design characteristics—designer's intent—by a point data set that must be mathematically manipulated to give results that can be compared to the drawing specification. The geometry of the real part must be reconstructed or derived from the data set. The derived geometry can be a boundary model of the part fitted to the data set or a feature such as an axis—resolved geometry in Y14.5.1—used as a proxy for the desired perfect geometry.

The derived geometry is defined in such terms that appropriate methods, procedures, and algorithms may be identified for the measurement task. In particular, a specific geometry-fitting criterion is to be selected based on functional requirements. Fitting algorithms such as least squares, maximum inscribed circle, minimum circumscribed circle, and mini-max can all yield different results depending on the nature of the measured part. This is of particular concern when dealing with parts that have significant form errors. Each fitting criterion may cause widely differing results and functional failure due to the manner in which the algorithm creates the derived geometry (Phillips, 1995). Precautions must be taken when selecting the fitting criterion, ensuring that the substitute geometry verifies the intended part function.

7.4.3 Conformance

Given a definition of the characteristic and its derived geometry, procedures are established to compare either one measured value to another or a measured value with its tolerance. Because there is uncertainty inherent in all measurement pro-

cesses, this step (step 4) is necessary to explicitly define the rules used in evaluating conformance. Guidance to be used in creating these rules can be found in ISO 14253-1, which provides decision rules for proving both conformance and nonconformance.

An important distinction must be made when deciding whether a workpiece conforms to specification. The tolerance zone created by the product specification and the conformance zone that results when uncertainty is considered are not identical. Furthermore, the size of the zones where conformance can be proved and where nonconformance occurs are subject to measurement uncertainty, a variable quantity.

Using a two-sided specification zone as an example, the effects of measurement uncertainty can be illustrated by Figure 7-10. The introduction of uncertainty into the measurement process reduces the specification limits. If the measurement is modeled as a measured value with uncertainty equally disposed about this value, it is apparent that the nonconformance region is also affected by uncertainty.

Taking the measurement and its associated uncertainty and moving it along a line representing the specification shows the results. In Figure 7-10(a), a measurement is acceptable as long as it never gets closer to the upper or lower specification limits than the boundaries created by the uncertainty analysis.

If the measured value becomes coincident with either of the specification limits [Figure 7-10(b)], the specification zone is reduced by one half the width

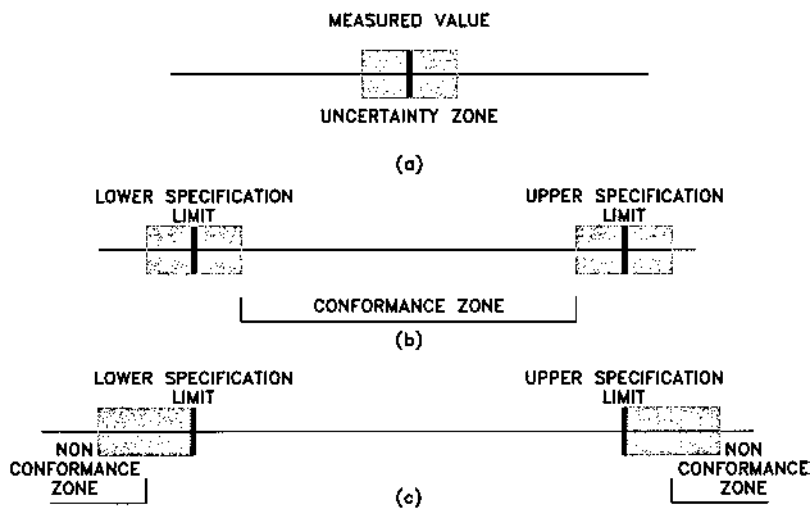


FIGURE 7-10 Measured values, uncertainty, and specification limits.

of the uncertainty zone. This width is the “expanded uncertainty” defined in ISO 14253-1 and creates the conformance zone.

When the measurement is outside the specification zone, another problem is encountered. As shown in [Figure 7-10\(c\)](#), the measurement is clearly out of specification when it is past the upper specification limit by a distance equal to the expanded uncertainty. However, if the measurement is near the limit and within a distance equal to the expanded uncertainty [[Figure 7-10\(b\)](#)], nonconformance cannot be proven. One can prove nonconformance only when the measurement is outside the specification limits by more than the expanded uncertainty.

As a consequence, there is a region where one cannot prove or disprove conformance of a measurement. This quandary arises when the uncertainty straddles the specification limit [[Figure 7-10\(b\)](#)]. To resolve the problem, the development team must reconcile two views of the product world. The first view is the designer’s, with sharply defined specification limits that describe design intent. The competing view is that of the metrologist, where conformance limits are established by the variables of the measurement process. In the latter world, there are no fixed limits and the variability is a function of the methods and procedures used in the measurement process.

The general rule stated in ISO 14253-1 is “the uncertainty of measurement always counts against the party who is providing the proof of conformance or nonconformance and therefore making the measurement.” The party proving conformance or nonconformance uses his or her actual uncertainty of measurement in making a decision.

7.4.4 Methods and Procedures

One must next choose an appropriate measurement method and select the necessary metrological equipment to implement it. Step 5 provides the theoretical structure within which the method should be selected. Step 6 provides the characteristics of the measuring instruments that will be used to implement the chosen method. The actual chain link encompasses only the metrological characteristics that influence the uncertainty of the measurement. There are also general design characteristics of the measuring equipment that are considered in assembling the instruments and selecting procedures to be followed. More is said about this aspect of planning in the next chapter.

7.4.5 Link to System of Units

Since interchangeable manufacturing and commerce require accurate measurements, the last step (step 7) defines the characteristics of a calibration process needed for the selected measuring equipment. This may include characteristics of the physical standards used in calibration procedures.

The calibration process provides traceability of all measurements made on the manufactured part to the national and international standards of length. Embedded in ISO documents is the requirement that this traceability be more than just a paper trail of documents that lead from the shop floor to a national laboratory. Statements of uncertainty are now required in each link of the traceability chain, with supporting evidence to show how these estimates were derived.

Since measurements on a manufactured product can encompass many types of geometric features, a variety of paths may be used to provide proof of traceability. These may range from the use of calibrated end standards, form standards, or mastered prototype parts. Establishing standards by the use of prototypes is exemplified by involute gearing. In some measurement applications in gear manufacture, a master gear is used as the standard. In the event that a calibrated master gear is used, a standardized procedure is necessary to calibrate the standard and demonstrate the unbroken traceability chain along with appropriate expressions of uncertainty.

7.5 SUMMARY

The continuing technological change in manufacturing precision has placed tremendous pressures on measurement and inspection processes. It was once possible to create gages of sufficient accuracy relative to the process they were intended to monitor so that no formal planning process was needed. However, with today's manufacturing processes now capable of precision that matches measurement capability, more thought is needed to provide useful information for decision making.

Three tools are presented in this chapter that can provide the foundation from which the new challenges may be met. Of a general nature are the introduction to the measurement model and the underlying statistical basis. An understanding of the place that each assumes in verifying whether a product conforms to specification is necessary before delving into the details of planning an actual measurement process. Of greater impact is the measurement planning process itself.

It has been a fairly typical occurrence to find process plans that included a single, ill-defined operation calling for "inspection." By default, the actual planning of the measurement process was left to discretion of any inspector that might be assigned the task. In past decades relatively coarse product tolerances allowed this haphazard method to yield economically viable accept–reject decisions. This is no longer the case.

A formal measurement planning process provides the control and consistency needed in the global environment, where manufacturers are being held to higher standards. The philosophy incorporated in the ISO 9000 series of standards demands a formal recognition of the elements of the product development pro-

cess, including the measurement planning stages. The presentation in this chapter emphasizes the basics that must be understood to meet the more stringent requirements now being forced on manufacturers by technological advances and market requirements.

REFERENCES

- Bennich, P., Seminar on Uncertainty, presented May 17, 1997, Gaithersburg, MD, under the auspices of the ASME.
- Bevington, P. R. and Robinson, D. K., *Data Reduction and Error Analysis for the Physical Sciences*, New York: McGraw-Hill, 1992.
- ISO, *Guide to the Estimation of Uncertainty in Measurement (GUM)*, 1st ed., 1993.
- ISO/14253-1: 1998, *Geometrical Product Specifications (GPS)—Inspection by Measurement of Workpieces and Measuring Equipment—Part 1: Decision Rules for Proving Conformance or Non-Conformance with Specification*.
- ISO/TS 14253-2: 1999, *Geometrical Product Specifications (GPS)—Inspection by Measurement of Workpieces and Measuring Equipment—Part 2: Guide to the Estimation of Uncertainty in GPS Measurement, in Calibration of Measuring Equipment and in Product Verification*.
- ISO/TR 14638, *Geometrical Product Specifications (GPS)—Masterplan*.
- Meadows, J. D., *Measurement of Geometric Tolerances in Manufacturing*, New York: Marcel Dekker, 1998.
- Phillips, S. D., Performance evaluations, in *Coordinate Measuring Machines and Systems*, J. A. Bosch, ed., pp. 149–152, New York: Marcel Dekker, 1995.
- Taylor, J. R., *An Introduction to Error Analysis*, Mill Valley, CA: University Science Books, 1982.